Volatility Clustering with Learning and Model Heterogeneity

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Contribution

We consider a standard Lucas economy with a single consumption tree, two agents, and an unobservable fundamental.

The agents have different parameter estimates for the structure of the economy. This generates *persistent* difference of beliefs.

The stock market volatility is significantly higher than the consumption volatility and strongly countercyclical: a low expected fundamental triggers high stock market volatility.

Using Malliavin calculus we show that the volatility clustering effect comes exclusively from the difference of opinion process.

Model

There is one perishable good, the numeraire, with price equal to unity. The economy is populated by two agents (A and B) who optimally consume an exogeneous dividend stream δ provided by one risky asset.

Agent A's perceptions of the processes driving the economy are

$$\frac{d\delta_t}{\delta_t} = f_{At}dt + \sigma_\delta dW^\delta_{At}$$
$$df_{At} = \lambda_A\left(\bar{f}_A - f_{At}\right)dt + \sigma_{Af}dW^f_{At} \quad (3)$$

Agent B's beliefs about the economy are

$$\frac{d\delta_t}{\delta_t} = f_{Bt}dt + \sigma_\delta dW^\delta_{Bt}$$
$$df_{Bt} = \lambda_B\left(\bar{f}_B - f_{Bt}\right)dt + \sigma_{Bf}dW^f_{Bt} \quad (4)$$

The expected growth rate (the fundamental) is not observable by the agents. The difference of beliefs ($\hat{g}$) arises because the agents have different parameters for the fundamental process:

$$d\hat{g}_t = \left[\mathcal{A} + (\lambda_A - \lambda_B)\hat{f}_{Bt} - \mathcal{B}\hat{g}_t\right]dt + \mathcal{C}d\hat{W}^\delta_{Bt} \quad (5)$$

where $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ are constants.

Solution Method

We solve the problem using the martingale approach of Karatzas, Lehoczky, and Shreve (1987) and Cox and Huang (1989). First, we compute the price of single-dividend paying stocks S^T_t

$$S^T_t = \mathbb{E}_t\left[\frac{\xi_T}{\xi_t}\delta_T\right] \quad (1)$$

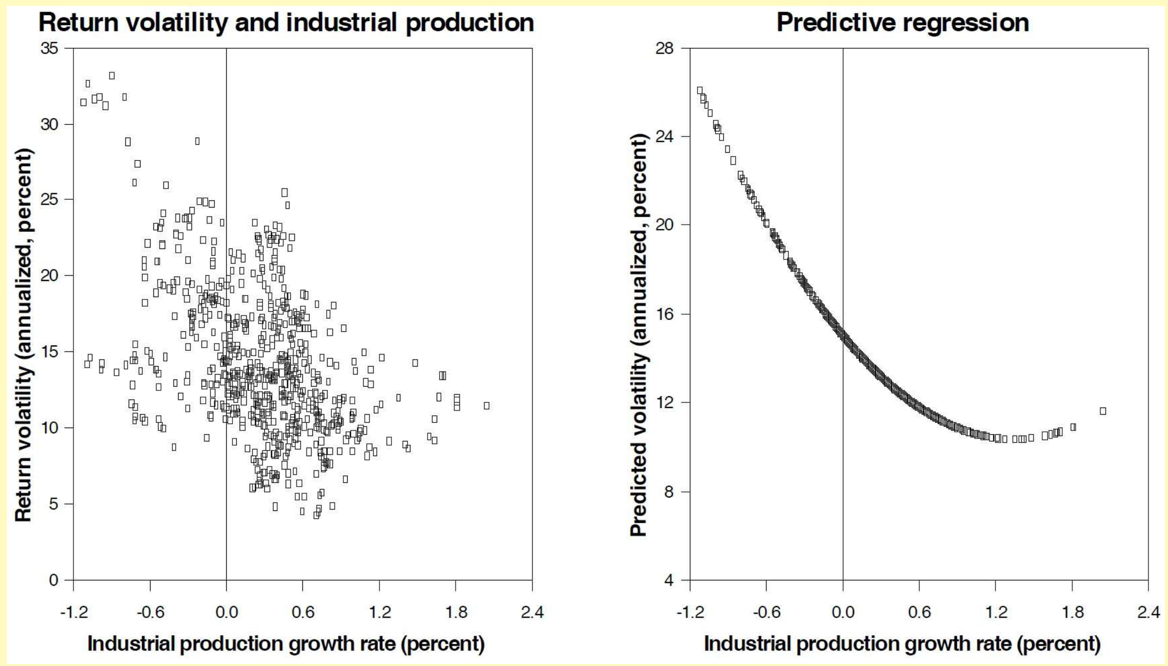
where ξ is the state-price density. The stock price is then computed as the sum of the single-payoff stocks, i.e.

$$S_t = \int_t^\infty S^u_t du \quad (2)$$

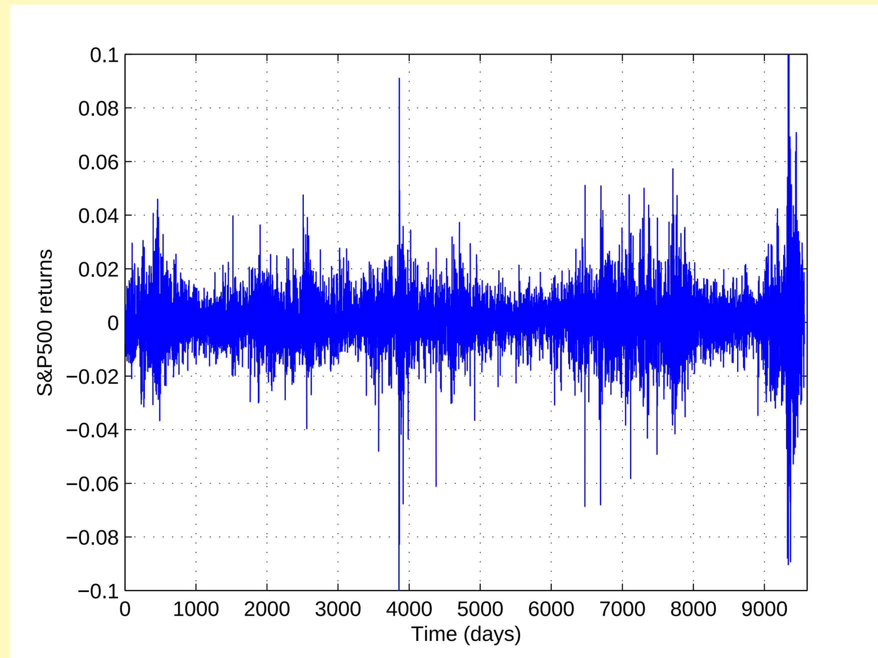
Since the structure of the model is affine quadratic, we use standard procedures to obtain the solution. We compute stock prices, the market price of risk, stock return volatility, and the optimal portfolio choice of the investors.

Motivation & Empirical Evidence

The stock market volatility is countercyclical and persistent:

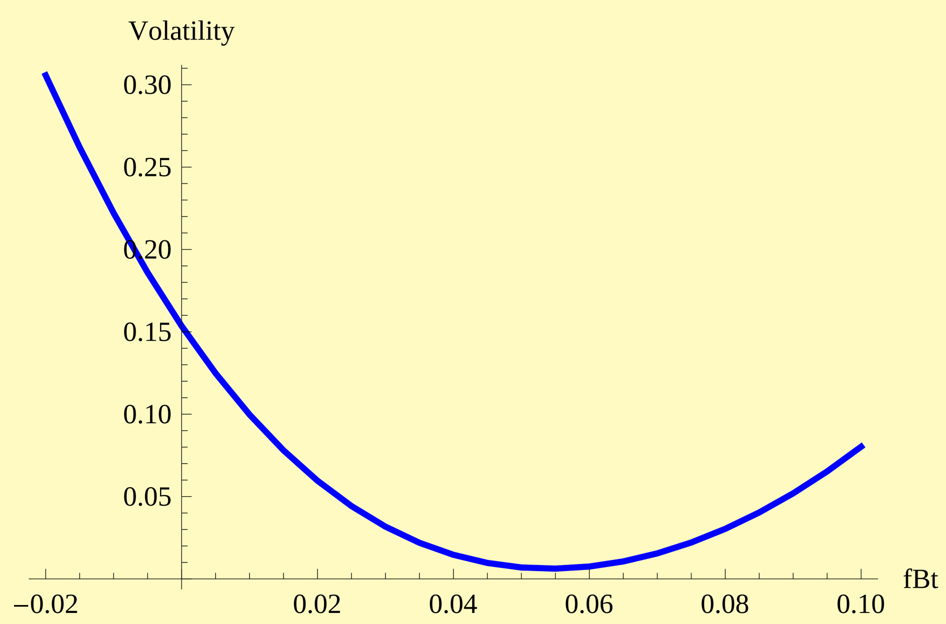


1) Stock return volatility against industrial production growth
Source: Mele (2008).

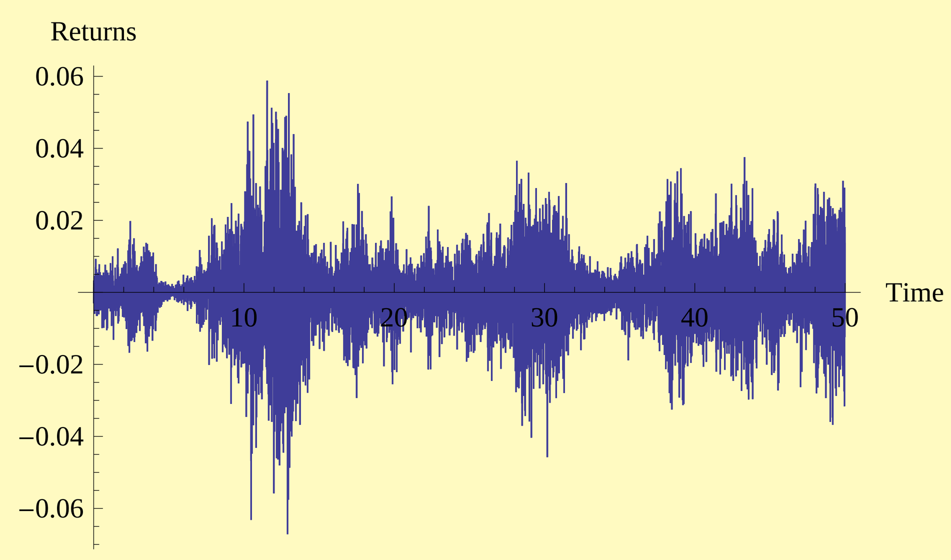


2) S&P Daily Returns 1970-2010.
Conditional variance fluctuates across time and is very persistent

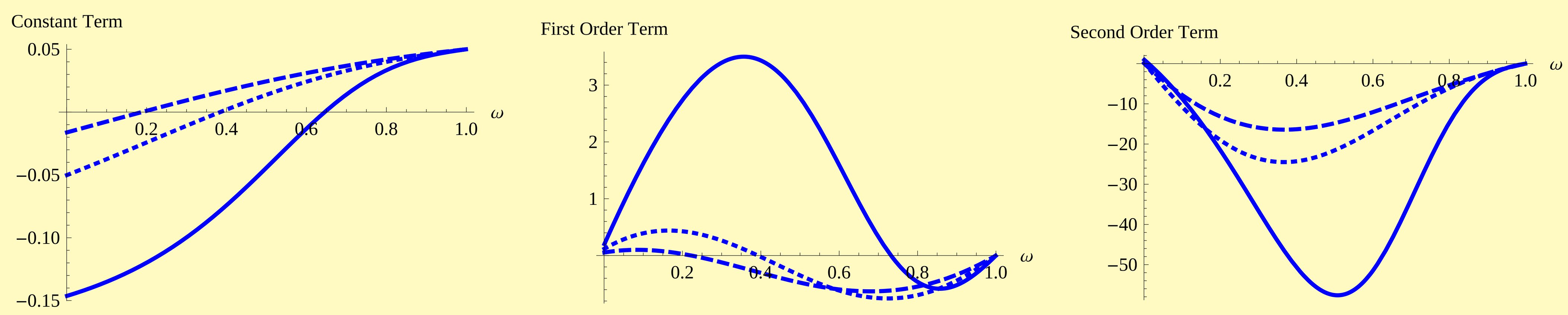
Results



3) Stock market volatility against expected growth rate
The volatility is countercyclical.



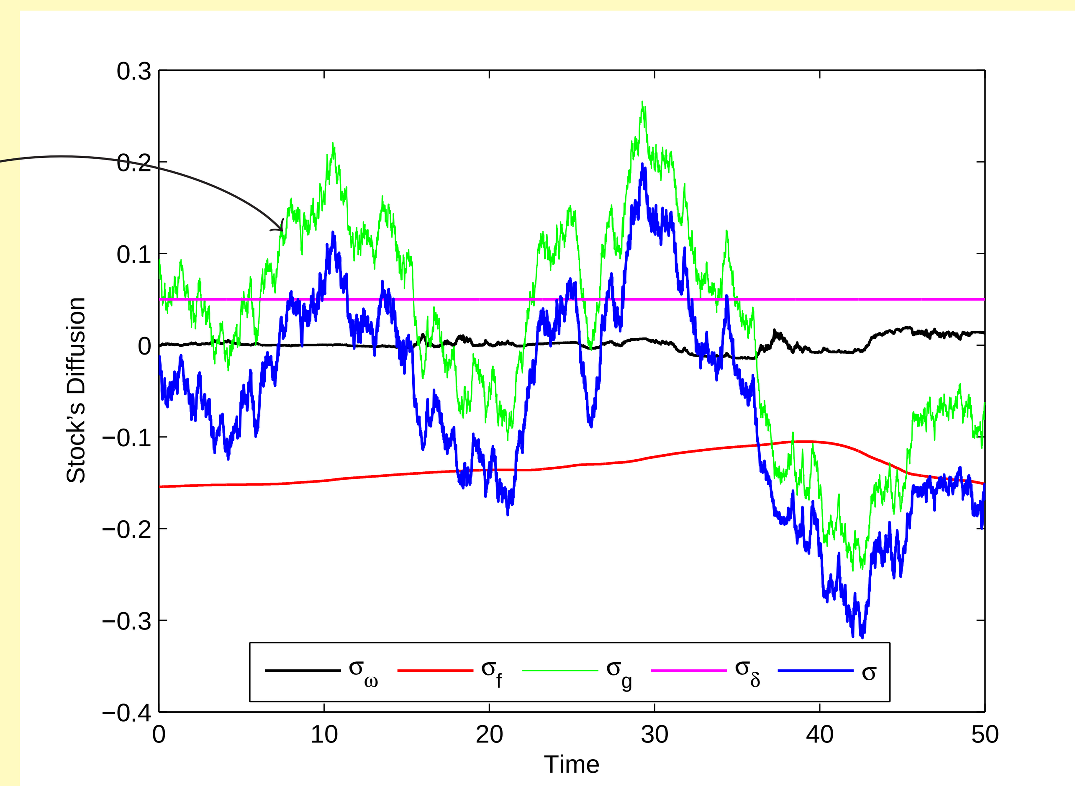
4) Daily returns simulated for 50 years
The volatility is clustering.



5) Second order Taylor decomposition of the stock return diffusion with respect to the expected growth rate $\hat{f}_{Bt}$, against the agent's A share of consumption, ω_t:
$$C_0(\omega_t) + C_1(\omega_t)\left(\hat{f}_{Bt} - \bar{f}\right) + C_2(\omega_t)\left(\hat{f}_{Bt} - \bar{f}\right)^2$$

6) Exact decomposition of the stock return volatility through Malliavin calculus:
$$\sigma = \sigma_\delta + \sigma_f + \sigma_\omega + \sigma_g$$

The difference in beliefs is driving the dynamics of the volatility.



References & Further Information

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