International Portfolio Choice and Relative Wealth Concerns*

Daniel Andrei†

July 2, 2010

Abstract

In a standard information based model à la Admati [1985] with private information, it is shown how a plausible social interaction between investors, namely the relative wealth concerns, might amplify an almost insignificant informational advantage and produce sizable home bias. The model considers a *quantitative* informational advantage, which is different of what has been postulated so far in the literature. The solution is found in closed form and is in line with the economic intuition. The home bias results are analyzed not only at country level, but as well at investor's level. It is shown that there is a cross-sectional variation within countries in the level of home bias.

*Preliminary and incomplete. I would like to thank Bernard Dumas and Philippe Bacchetta for several stimulating conversations. I would also like to acknowledge comments from Kenza Benhima, Gustavo Manso, Alexandru Popescu, Hélène Rey, René Stulz and seminar participants at Gerzensee 2010.

†Swiss Finance Institute, University of Lausanne - Institute of Banking and Finance, dandrei@unil.ch, www.danielandrei.net.

Contents

1 Introduction

The objective of this work is to show in an international setup how a plausible social interaction between investors, namely the relative wealth concerns, might amplify an almost insignificant informational advantage and produce sizable home equity bias. Differently from existing literature, the initial informational advantage is that in each country more investors have private information about domestic assets than for foreign assets, while the *precision of the information remains the same*. Thus, it is more a quantitative than a qualitative informational advantage. This informational advantage is so small, that in an Admati [1985] type model with private information it will produce an almost insignificant local bias towards domestic assets.

However, when informed investors exhibit "keeping up with the Joneses" (KUJ) preferences, the initially insignificant effect of the informational advantage is greatly amplified and investors tilt massively their portfolios towards local assets. This effect appears because, when they have KUJ preferences, the investors tend to imitate the others and therefore if in their own country there are more investors informed about the domestic asset, all of them will increase their holding in that asset.

The setup assumes the following. There is a continuum of investors, situated in two equal-sized countries. In each country there is a risky asset. Some of the investors have private information about one or several assets. There is a noisy supply of the two risky assets, preventing uninformed investor to fully understand the private information of the informed investors. All the investors are free to trade in international financial markets.

As previously stated, it is assumed that in each country more investors have private information about domestic assets than for foreign assets, while the precision of the information remains the same. This hypothesis is different from the one concerning the precision of the private information and appears more plausible, for two main reasons. *First,* because under the standard model without wealth concerns this will produce a minimal effect on portfolio holdings. That is, the Admati [1985] type model with private information is silent about where the informed agents are. What it matters for portfolio choice is only the proportion of privately informed agents. However, we will see soon that the relative wealth concern effect will take into account the location of informed investors. And if more investors are informed about the domestic asset, this will translate into a tilt of the portfolio towards that domestic asset. *Second,* it is an obvious result that if investors have more precise information about the domestic assets, they will be more home biased. For this purely mathematical and straightforward result see Gehrig [1993]. It is not the aim of this work to assume that the distance has a role in explaining the information advantage of home investors, i.e., if the distance has an influence on the precision of information. Although some recent literature tend to confirm this view[1], advances in information and communication technologies will make this assumption hard to sustain. Therefore, it would be a lot more challenging to obtain home equity bias without assuming this kind of information advantage.

This *quantitative informational advantage* will have no effect on optimal portfolio holdings of the individual investor. However, once the assumption of relative wealth concerns is taken into account, investors tend to overweight domestic assets in their portfolio. Neither of the informational advantage or the KUJ assumptions alone will not produce any relevant home bias. The effect will appear only when this two assumptions are considered together.

An additional contribution is that the model is solved in closed form and happens to have a very intuitive solution. As it will be shown in next sections all the parameters can be interpreted easily and have a powerful economic significance. The home bias is analyzed not only at country level, but as well at investor's level. It will be shown that there is a cross-sectional variation within countries of the level of home bias, consistent with recent empirical findings, such as Hau and Rey [2008].

Several authors have used consumption externalities to explain home bias[2]. In this paper I propose a somewhat new mechanism, namely the interaction between a small quantitative informational advantage and the

[1] see, for example, Bae et al. [2008]

[2] see, for example, Covrig et al. [2004], Gomez Lopez et al. [2009], Lauterbach and Reisman [2004], Shore and White [2002].

relative wealth concerns. The aim is to explain how social interaction might have an impact on the optimal portfolio holdings and produce a domestic investment preference.

The rest of the work is organized as follows. Section 2 solves the model in the benchmark case, when there are no relative wealth concerns. Section 3 deals with the relative wealth concerns case. The results in terms of average portfolio holdings and home bias are presented in section 3.2. Finally, section 4 concludes and make some plans for further work. Details of the computations are presented in the appendix.

2 A Benchmark Model

The benchmark model is build as usual in standard information models, except that it is assumed that home investors have an informational advantage related to domestic assets when compared to foreign investors.

This informational advantage is somewhat different of what is considered in the existing literature. Usually, the starting point is that investors have more precise information about the domestic assets than the foreign ones[3]. Instead of that, I will assume that in each country more investors have private information about domestic assets than for foreign assets, *while the precision of the information remains the same*. This hypothesis is more appealing than the one concerning the precision of the private information, for two main reasons.

First, because under the current benchmark model, this will produce a minimal effect on portfolio holdings, as it will be shown in the next section. That is, the Admati [1985] type model with private information is silent about *where* the informed agents are. What it matters is only the proportion of privately informed agents. However, we will see soon that the relative wealth concern effect will take into account the location of informed investors. And if more investors are informed about the domestic asset, this will translate into a tilt of the portfolio towards that domestic asset.

[3]See, for instance, Gehrig [1993], or Brennan and Cao [1997].

Second, it is an obvious result that if investors have more precise information about the domestic assets, they will be more home biased. For this purely mathematical and straightforward result see Gehrig [1993]. However, given the advance of the information technology, it is not easy to state that the distance has an influence on the precision of the information. A more realistic view will be that the precision of the information is the same, no matter the location, and something else (e.g. relative wealth concerns) has an influence on the optimal portfolio holdings.

2.1 The Model

There is a continuum of investors indexed by $i \in [0, 1]$, living in two equal-sized countries, Home and Foreign. Without loss of generality, country Home (H) is defined over the interval $[0, 1/2]$ and country Foreign (F) is defined over the interval $(1/2, 1]$. Investors are characterized by exponential utility function defined over final wealth with common coefficient of absolute risk aversion τ, $u(W_i) = -\exp[-\tau W_i]$. The initial wealth of each investor i is normalized to 0, without loss of generality. Trading take place at date 0 and consumption takes place at date 1.

In each country there is one risky asset; that is, asset 1 in H and asset 2 in F. The payoffs of the two risky assets are represented by a 2×1 normally distributed random vector $X = [X_1 \ X_2]^\top$ with mean $[\mu_x \ \mu_x]^\top$ and precision matrix $\pi_x I_2$, where I_2 stands for the 2×2 identity matrix. As usual in rational expectations models, there is an aggregate supply of the risky assets, a normally and independently distributed random vector $Z = [Z_1 \ Z_2]^\top$ with mean $[\mu_z \ \mu_z]^\top$ and precision matrix $\pi_z I_2$. Therefore, the equilibrium will not be fully revealing due to the presence of noisy supply. Additionally, there is a riskless asset in perfectly elastic supply with a price and payoff normalized to 1. All the three assets are perfectly available for trading internationally.

Each investor has the chance to receive some private information about one or two risky assets. Thus, in each country there are four types of investors.The first type, $i(1,2)$, are the fully informed, who receive both private signals. The second type, $i(1)$, receive the private signal about the payoff of asset 1 only. The third type, $i\,(2)$, receive the private signal about the payoff of asset 2 only. Finally, the fourth type, $i\,(0)$, have no private information about either asset. In what follows I will name with i_1 the event "investor i receives private information about asset 1" and with i_2 the event "investor i receives private information about asset 2". The vector of private signals has the form $Y_i = X + \varepsilon_i$, where $\varepsilon_i = [\varepsilon_{i1}\ \varepsilon_{i2}]^\top$ is normally and independently distributed with mean zero and precision matrix $\pi_\varepsilon I_2$.

A global proportion of $\lambda \in [0,1]$ investors have information about the asset 1 and an equal proportion of λ investors have information about the asset 2. There is a correlation between investor's location and the probability of receiving private information about the risky assets. That is, the probability of receiving private information is *dependent* on investor's location. Accordingly, for a given $0 \leq \omega \leq \min[\lambda, 1-\lambda]$:

- for an investor situated in the Home country, the event i_1 has probability $\lambda + \omega$ and the event i_2 has probability $\lambda - \omega$;
- for an investor situated in the Foreign country, the event i_1 has probability $\lambda - \omega$ and the event i_2 has probability $\lambda + \omega$.

Therefore, eight types of investors are present in the model with the weight of each investor type shown in Table 1.

	Country H	Country F
$i^{(\cdot)}(1,2)$	$\frac{\lambda^2-\omega^2}{2}$	$\frac{\lambda^2-\omega^2}{2}$
$i^{(\cdot)}(1)$	$\frac{(\lambda+\omega)(1-\lambda+\omega)}{2}$	$\frac{(\lambda-\omega)(1-\lambda-\omega)}{2}$
$i^{(\cdot)}(2)$	$\frac{(\lambda-\omega)(1-\lambda-\omega)}{2}$	$\frac{(\lambda+\omega)(1-\lambda+\omega)}{2}$
$i^{(\cdot)}(0)$	$\frac{(1-\lambda)^2-\omega^2}{2}$	$\frac{(1-\lambda)^2-\omega^2}{2}$

Table 1: Investor types and their respective weights in the total population. For example, there is a $(\lambda+\omega)(1-\lambda+\omega)/2$ proportion of type $i^H(1)$ investors. All the weights must sum up to one.

Notice that if $\omega = 0$, there is independence between investor's location and the probability of receiving information about a given asset. If $\omega > 0$, this means that investors have more chances to obtain private information about the domestic asset's payoff than about the foreign asset's payoff. The aforementioned probabilities can be written in the following form

$$\begin{aligned} \mathbb{P}\left[\text{event } i_1 \mid \text{Home investor}\right] &= \lambda + \omega \\ \mathbb{P}\left[\text{event } i_2 \mid \text{Home investor}\right] &= \lambda - \omega \\ \mathbb{P}\left[\text{event } i_1 \mid \text{Foreign investor}\right] &= \lambda - \omega \\ \mathbb{P}\left[\text{event } i_2 \mid \text{Foreign investor}\right] &= \lambda + \omega \end{aligned} \tag{1}$$

It can be verified by the Law of total probability that

$$\mathbb{P}\left[i_1\right] = \mathbb{P}\left[i_1 \mid H\right]\mathbb{P}\left[H\right] + \mathbb{P}\left[i_1 \mid F\right]\mathbb{P}\left[F\right] = \lambda \tag{2}$$

and, in the same way, $\mathbb{P}\left[i_2\right] = \lambda$.

Let $S_{i(\cdot)}^{\top} = \left[S_{i(\cdot)}^{1} \; S_{i(\cdot)}^{2}\right]$ denote the number of shares of the risky assets bought by agent $i\left(\cdot\right)$. Assuming zero initial wealth, the final wealth of investor $i_{(\cdot)}$ is $W_i = S_{i(\cdot)}^{\top}\left(X - P\right)$, where $P = \left[P_1 \; P_2\right]^{\top}$ denotes the price vector of the risky assets. As is customary in the literature, the equilibrium price is postulated as a linear function of the average signals and the aggregate stock supply, such that $P = a + cX - cBZ$, with

$$a = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \; c = \begin{bmatrix} c11 & c12 \\ c21 & c22 \end{bmatrix} \text{ and } B = \begin{bmatrix} B11 & B12 \\ B21 & B22 \end{bmatrix} \tag{3}$$

I define $Q \equiv c^{-1}\left(P - a\right) = X - BZ$ the normalized price signal, informationally equivalent to P. Solving for equilibrium requires conjecturing the trading strategy of all the agents. Thus, it is assumed that type $i\left(1,2\right)$ agent's trading strategy is $S_{i(1,2)} = \alpha + \beta Y_i - \delta Q$, type $i\left(1\right)$ agent's trading strategy is $S_{i(1)} = \phi_1 + \eta_1 Y_i - \kappa_1 Q$, type $i\left(2\right)$ agent's trading strategy is $S_{i(2)} = \phi_2 + \eta_2 Y_i - \kappa_2 Q$ and type $i\left(0\right)$ agent's trading strategy is $S_{i(0)} = \zeta - \nu Q$. The coefficients in the demand functions are either 2×1 vectors (α, ϕ_1, ϕ_2 and ζ), either 2×2 matrices (the others). Note that the second column of η_1 and the first column of η_2 are zero, since type $i\left(1\right)$ agents do not receive any private signal about asset 2 and type $i\left(2\right)$ agents do not receive any private signal about asset 1.

The average portfolios for each investor type is obtained by integration. Aggregating all the average portfolios and imposing market clearing leads to

$$\Gamma_1 + \Gamma_2 X - \Gamma_3 Q = Z, \tag{4}$$

with

$$\begin{aligned}
\Gamma_1 &= \left(\lambda^2 - \omega^2\right)\alpha + \lambda\left(\lambda - \lambda^2 + \omega^2\right)(\phi_1 + \phi_2) + \left((1-\lambda)^2 - \omega^2\right)\zeta \\
\Gamma_2 &= \left(\lambda^2 - \omega^2\right)\beta + \lambda\left(\lambda - \lambda^2 + \omega^2\right)(\eta_1 + \eta_2) \\
\Gamma_3 &= \left(\lambda^2 - \omega^2\right)\delta + \lambda\left(\lambda - \lambda^2 + \omega^2\right)(\kappa_1 + \kappa_2) + \left((1-\lambda)^2 - \omega^2\right)\nu
\end{aligned} \tag{5}$$

Since it was assumed before that $Q = X - BZ$, it is easy to verify that $B^{-1} = \Gamma_2$. Then, after writing the optimality conditions for each investor type, it follows that β, η_1 and η_2 have simple forms. They are provided in Appendix section A.1. After replacing them in Γ_2, it turns out that B is simply equal to $\frac{\tau}{\lambda\pi_\varepsilon}I_2$. This corresponds to Lemma 3.2 in Admati [1985].

Once solving for B, it is straightforward to obtain solutions for the other coefficients. These are provided in Appendix section A.1.

2.2 Home Bias

The total portfolio holdings of the Home investors and the resulting home bias, as a function of the asymmetry parameter ω, are exposed in Figure 1. I compute the home bias for the Home investors using the following measure

$$HB = \text{domestic holdings} - \frac{\text{home capitalization}}{\text{world capitalization}} \tag{6}$$

This measure of home equity bias is described by Sercu and Vanpee [2007]. More specifically, an international CAPM predicts that rational investors should hold the world market portfolio of risky securities. The home bias measure is equal to the difference between the observed proportion of domestic holdings and the domestic weight in the world market capitalization. Following the same study of Sercu and Vanpee [2007], the equity home bias ranges between 32 percent and 99.7 percent, with all countries holding significantly home-biased equity portfolios.

In the benchmark model, I compute the total portfolio holdings for the Home investors, as a function of the asymmetry parameter ω. This is shown in the left panel of Figure 1, together with the calibration used. If more investors receive private information for the domestic asset than for the foreign asset ($\omega > 0$), then the aggregate investment in the domestic asset will be larger than the aggregate investment in the foreign asset. This difference in portfolio weights increases as ω gets larger. As a result, there will be an almost insignificant home equity bias, shown in the right panel of Figure 1. Similar results are obtained for the Foreign investors (in their case, asset 2 is the domestic one).

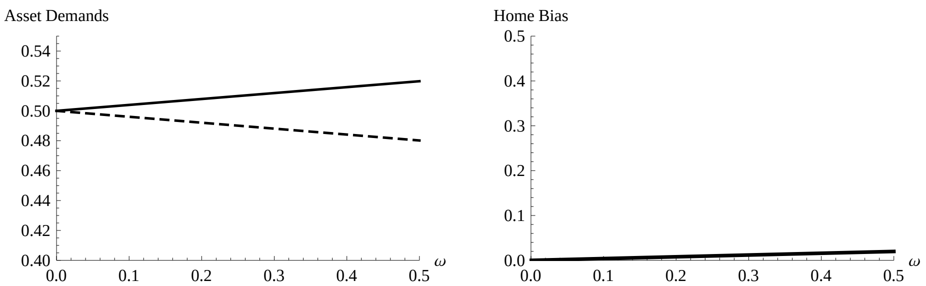

Figure 1: Total portfolio holdings and home bias for the Home investors, as a function of the asymmetry parameter ω. The left panel shows the total portfolio holdings (solid line for the domestic asset and dashed line for the foreign asset). The right panel shows the resulting home bias, computed as in (6). The calibration used is $\lambda = 0.5$, $\pi_x = \pi_z = \pi_\varepsilon = 20$, $\tau = 3$, $\mu_x = 1.3$, $\mu_z = 1$.

These results are not at all surprising. They show that if there are more investors informed about the domestic asset, the aggregate portfolio demand for this asset will be larger that for the foreign asset. However, after a careful examination of the optimal demands for the different investor types, we realize that none of them depends on the asymmetry parameter ω. That is, if an investor has private information about asset 1, it does not matter if he is located in the Home country or in the Foreign country, he will have *exactly the same* optimal demand. And the argument is the same for all other investor types.

This means that the home bias obtained so far is purely a *composition* *result*. Home investors will take on aggregate more of the domestic asset only because a higher proportion of them have private information about that asset. Still, their individual demand will remain the same. However, recent empirical findings[4] show that the geographical distance might have an impact on portfolio holdings of informed investors, which is not the case in this benchmark model.

Therefore, in such a standard setting, imposing a quantitative informational advantage of domestic investors will not make any change in their optimal individual demands. They will be equal to the standard demands of an standard Admati [1985] type model. The analysis must be extended to a more precise social level. We need to consider communities of investors in each country. In each community there should be some social interaction which makes that the location of investors has an influence on his portfolio holdings. This analysis will be developed in the next section.

3 Relative Wealth Concerns

In the last section I computed the equilibrium for the benchmark case. It turns out that the demand functions found in the benchmark model will be useful when there is some social interaction between the investors, e.g. relative wealth concerns. The additional hypothesis is that, when they are informed, investors care about the average wealth of other *domestic* investors who have information regarding the same assets as them. It will be shown that if there is a small informational advantage, ($\omega > 0$), the relative wealth concern effect will generate a sizable amount of local bias.

Intuitively, the consumption externality introduced in this section works as follows. Investors informed about one of the assets belong all to some community in their own country. They meet together regularly and they get enjoyment from talking about the market, as in Hong et al. [2004]. Then, it is assumed that they care about the average wealth of the other investors from their community. This hypothesis is easily sustainable, since people

[4] Portes and Rey [2005], Feng and Seasholes [2004]

living in one country want to have a standard of living similar with their neighbors, social group, etc. As a result, the social interaction between them will amplify whatever aggregate local preference is.

Finally, it is assumed that only informed investors have relative wealth concerns. If we think that the most likely investors to possess private information should be professional fund managers, it is obvious that "keeping up with the Joneses" preferences are more relevant for them than for the uninformed ones. This should result from simple mechanisms such as benchmarking or career concerns.

3.1 The Model

There is one main modification with respect to the benchmark model from section 2.1. It is assumed that if an agent has some private information about at least one of the assets, his preferences will exhibit relative wealth concerns. That is, informed agents have preferences of the form $\mathrm{E}\left[u\left(W_i, \bar{W}_i\right)\right]$, where W_i denotes the agent's terminal wealth, and $\bar{W}_i$ denotes the value of a reference portfolio corresponding to agent i. Specifically, I assume that informed agent's utility function is given by

$$u\left(W_i, \bar{W}_i\right) = -\exp\left[-\tau\left(W_i - \frac{\gamma}{1+\gamma}\bar{W}_i\right)\right]. \tag{7}$$

The parameter γ captures the extent of the consumption externality, i.e., how much agent i cares about other agent's wealth. Thus, a investor's satisfaction with his own consumption depends on how much others are consuming. This functional form has been used by Garcia and Strobl [2009] to show how the relative wealth concerns affect investors' incentives to acquire information. The utility function is increasing and concave in W_i, with a coefficient of absolute risk aversion of τ, thus satisfying the usual conditions with respect to an agents own consumption.

The reference portfolio, $\bar{W}_i$, will be different for each investor type. In order to understand the differences, some extra notation is needed at this point. Denote by θ^H the aggregate demand of H investors and θ^F the aggregate demand of F investors. The demands for different H investors are denoted by $\theta^H_{i(\cdot)}$ and the demands for different F investors are $\theta^F_{i(\cdot)}$. We have the same categories of investors as in the benchmark case: H investors with both private signals, $i^H(1,2)$, F investors with both private signals, $i^F(1,2)$, H investors with a private signal for asset 1 only, $i^H(1)$, and so on.

I consider that if an agent has some private information regarding one of the assets $j \in \{1,2\}$, then he belongs to the *domestic* community of agents who have information about the same asset. For example, if an investor is of the type $i^H(1,2)$, then he will belong to the community of investors from the Home country informed either about asset 1, either about asset 2. This form of social interaction is in line with recent empirical findings by Hong et al. [2004], Feng and Seasholes [2004] and others[5], who have emphasized the importance of peer-group effects in the investment choice of the individuals.

Once the reference portfolio for each investor type is build, I can solve for the equilibrium portfolio holdings in the usual way. The following lemma is required for the computation of the equilibrium. It is a standard result on multivariate normal variables (see, e.g., Rahi and Marín [1999] and the reference therein for a proof):

Lemma 1. *Let $z \in \mathbb{R}^n$ be a normally distributed vector with mean μ and covariance matrix Σ. If $I - 2\Sigma A$ is positive definite, then $E\left[\exp\left(X'AX\right) + b'X\right]$ is well-defined and given by*

$$|I - 2\Sigma A|^{-1/2}\left[b'\mu + \mu' A\mu + \frac{1}{2}\left(b + 2A\mu\right)'\left(I - 2\Sigma A\right)^{-1}\left(b + \Sigma A\mu\right)\right]$$

where $A \in \mathbb{R}^{n \times n}$ is a symmetric matrix and $b \in \mathbb{R}^n$ is a vector. In a simpler form, if $z \sim N(0, \Sigma)$ then

$$E\left[e^{z'Fz + G'z + H}\right] = |I - 2\Sigma F|^{-\frac{1}{2}}\, e^{\frac{1}{2}G'(I - 2\Sigma F)^{-1}\Sigma G + H}$$

For the computation of the equilibrium, I will start by assuming that the demand function of each investor type is a linear combination of the demand functions found previously in the benchmark case. For example, the demand

[5] Grinblatt and Keloharju [2001], Brown et al. [2008], Ivkovic and Weisbenner [2005]

function for the $i^H(1,2)$ type investors is defined as

$$\theta^H_{i(1,2)} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} S_{i(1,2)} + \begin{bmatrix} \psi_H & 0 \\ 0 & \psi_F \end{bmatrix} S_{i(0)}, \tag{8}$$

with the parameters ψ_H and ψ_F to be determined. Intuitively, the $i^H(1,2)$ type investors move away from the benchmark optimal demand as $\gamma > 0$. Indeed, it will be checked later that $\psi_H = \psi_F = 0$ if the parameter γ is equal to zero.

The solution method starts by assuming again that the price is a linear function of the average signals and the aggregate stock supply, as in section 2.1. The matrix B will have exactly the same solution, $B = \frac{\tau}{\lambda \pi_\varepsilon} I_2$. The vector a and the matrix c will have a different form, more complicated than in the benchmark case, and available upon request. Finally, the rest of the coefficients are obtained by imposing market clearing. Postulated demands for all the investors and some details of the solution are shown in Appendix section A.2.

3.2 Results

3.2.1 Average Individual Demands

The same calibration is considered as in the benchmark case, except that now I fix $\omega = 0.2$, i.e., it is assumed that 70% of the investors from the Home country have information about asset 1, 30% about asset 2 and vice versa for the Foreign country.

In Figure 2 I show how the new coefficients depend on the parameter γ. One can see that ψ_H is increasing in γ (left panel, solid line). This means that if informed investors of the type $i^H(1,2)$ are more concerned about others' wealth, they will increase the weight in the domestic asset. More interesting, as γ becomes larger, ψ_F is decreasing, taking negative values. If informed investors of the type $i^H(1,2)$ are more concerned about others' wealth, *they will take less of the foreign asset.* There is a difference between ψ_H and ψ_F only because $\omega > 0$, which suggests that there is now a community effect in the portfolio holdings. That is, an individual Home informed investor of the type $i^H(1,2)$ will exhibit now home equity bias, which was not the case in the benchmark model.

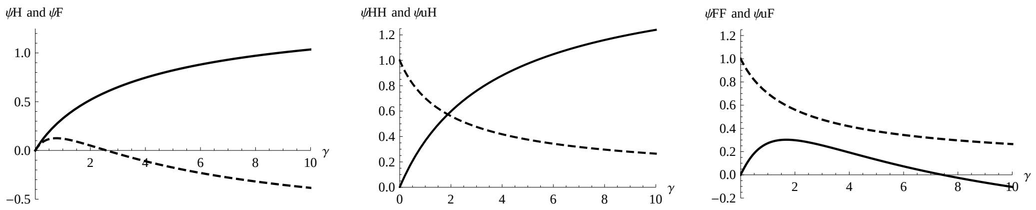

Figure 2: Parameters ψ_H, ψ_F, ψ_{HH}, ψ_{uH}, ψ_{FF} and ψ_{uF} as functions of the parameter γ. In each plot, the solid line is for the first parameter, and the dashed line is for the second one. The calibration used is $\lambda = 0.5$, $\omega = 0.2$, $\pi_x = \pi_z = \pi_\varepsilon = 20$, $\tau = 3$, $\mu_x = 1.3$, $\mu_z = 1$.

Let us consider now a Home investor of type $i^H(1)$. The results are in the middle panel of Figure 2. As γ gets larger, he will massively increase his holdings in the domestic asset, and decrease his holdings in the foreign asset. This investor was already home equity biased in the benchmark case, because he had private information only about asset 1. His home equity bias will increase now dramatically as γ gets larger.

For the Home investor of type $i^H(2)$, we know that in the benchmark case he was foreign equity biased. He will continue to be in this case as well (right panel, Figure 2). However, if γ gets large, that is, if he cares too strongly about the average wealth of the investors from this community, at some point he will start to decrease the holdings in the foreign asset, and thus reduce his foreign equity bias.

Note that if $\gamma = 0$, then ω has no influence on the optimal demands, i.e., investors will have exactly the same portfolios as in the benchmark case. Additionally, the parameter γ will make investors modify their portfolios only if the parameter ω is larger than 0. Neither of the information asymmetry or the relative wealth concerns by itself will make investors have different portfolios if they have different locations. The effect of the relative wealth concerns is there only because initially there is an informational asymmetry.

To better understand the implications for international portfolio holdings, I analyze as before the average portfolios of all informed investor types. This is done in Figure 3. The graphs represents the average portfolio holdings for each of the three types of the Home informed investors. Starting with the left panel - corresponding to type $i^H(1,2)$ investors, it is noticed that, as the parameters ψ suggested, the investors increase their holdings in the domestic assets and decrease them for the foreign asset. This result is more pronounced as γ gets larger, and only because $\omega > 0$.

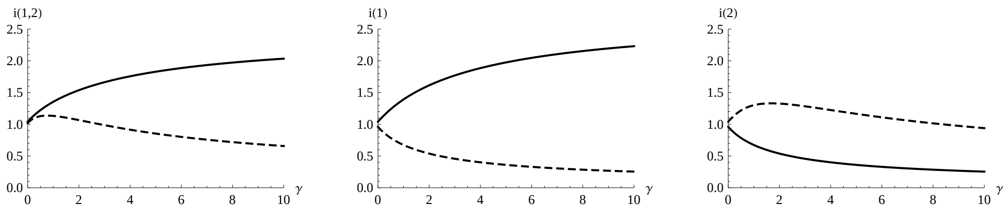

Figure 3: Average portfolio holdings for each of the four types of the Home investors. Solid lines represent holdings in asset 1 and dashed lines holdings in asset 2. The graphs will be the same for the Foreign investors, except that asset 2 becomes the domestic asset for them. The calibration used is $\lambda = 0.5$, $\omega = 0.2$, $\pi_x = \pi_z = \pi_\varepsilon = 20$, $\tau = 3$, $\mu_x = 1.3$, $\mu_z = 1$.

The middle panel shows the average portfolio holdings of the $i^H(1)$ type investors. Recall that these investors have private information about the domestic asset only. The same interpretation applies. For the foreign asset position, it is not surprising that the agents will decrease their position as the parameter γ increases, and they will increase their home asset position.

The average portfolio holdings of the $i^H(2)$ type investors are described in the right panel. Now the investors have information only about asset 2. This will make them decrease their domestic asset position as γ increases. However, even if they have information about the asset 2, they might decrease their position in the foreign asset as γ becomes large (see dashed line).

Note that all the plots confirm the intuition from section 3.1 and Figure 2. Additionally, the model seems to have nice implications for the cross-section of portfolio holdings (across investor types). If the parameter γ is zero we observe that there is an almost insignificant difference between the holdings of home and foreign assets. The difference is increased substantially when $\gamma > 0$.

3.2.2 Home Bias

The average portfolio holdings for all the investors in the Home country are shown in Figure 4. These are obtained by making the sum of average holdings over investor types, using the corresponding weight for each type. It is now easy to see that if γ increases, the average Home investor will increase the holding in the domestic asset and decrease the holding in the foreign asset. Note that for $\omega = 0$ the parameter γ has no effect. Thus, the relative wealth concerns might be an important factor in explaining the home equity bias.

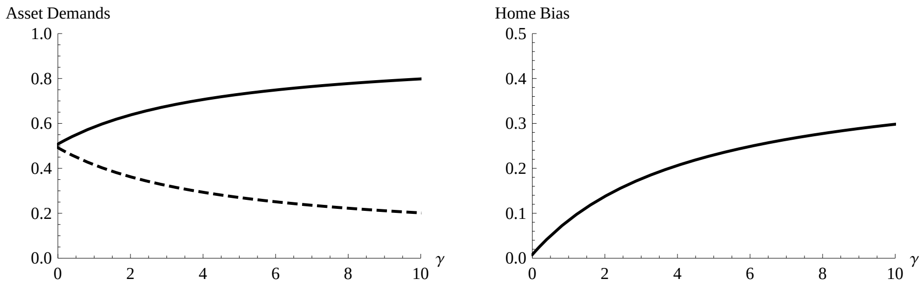

Figure 4: Total portfolio holdings and home bias for the Home investors, as a function of the parameter γ. The left panel shows the total portfolio holdings (solid line for the domestic asset and dashed line for the foreign asset). The right panel shows the resulting home bias, computed as in (6). The calibration used is $\lambda = 0.5$, $\omega = 0.2$, $\pi_x = \pi_z = \pi_\varepsilon = 20$, $\tau = 3$, $\mu_x = 1.3$, $\mu_z = 1$.

In Figure 5 I proceed to a decomposition of the home bias by investor type in the Home country. A similar analysis with identical plots could be done for the Foreign country. The fully informed investors, $i^H(1,2)$, exhibit no home bias when $\gamma = 0$, but then they start to bias substantially their portfolios towards home assets when $\gamma > 0$. This confirms the plots of portfolio holdings from section 3.2.1. The investors having private information only about asset 1, $i^H(1)$, start already with some amount of home bias when $\gamma = 0$ (this is not surprising, since they have information only about the home asset) and amplify it as γ increases. Note that in the case $\gamma = 0$ the amount of home bias is minimal. The investors having private information only about asset 2, $i^H(2)$, are initially foreign biased and they increase the foreign bias as γ increases, confirming the intuition form section 3.2.1.

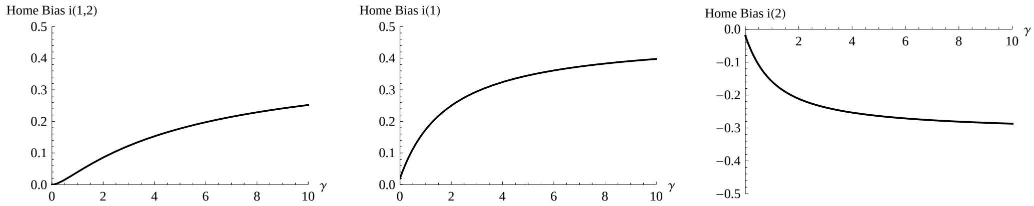

Figure 5: Home bias at the investor's type level. The calibration used is $\lambda = 0.5$, $\omega = 0.2$, $\pi_x = \pi_z = \pi_\varepsilon = 20$, $\tau = 3$, $\mu_x = 1.3$, $\mu_z = 1$.

4 Conclusions

I have shown how a plausible social interaction, namely relative wealth concerns, might amplify an almost insignificant quantitative informational advantage and produce sizable home bias. The model is solved in closed form and intuitive results are discussed. The home bias results are analyzed not only at country level, but as well at investor's level. It is shown that there is a cross-sectional variation within countries in the level of home equity bias.

A Appendix

A.1 Solution for the Benchmark Model

After finding $B = \frac{\tau}{\lambda\pi_\varepsilon} I_2$, consider the normally distributed vector

$$\xi = [X_1 \ X_2 \ Y_{i1} \ Y_{i2} \ Q_1 \ Q_2]^\top$$

with mean $\mu_\xi = \left[\mu_x \ \mu_x \ \mu_x \ \mu_x \ \mu_x - \frac{\tau}{\lambda\pi_\varepsilon}\mu_z \ \mu_x - \frac{\tau}{\lambda\pi_\varepsilon}\mu_z\right]^\top$ and variance-covariance matrix

$$\sigma_\xi = \begin{bmatrix} \frac{1}{\pi_x} & 0 & \frac{1}{\pi_x} & 0 & \frac{1}{\pi_x} & 0 \\ 0 & \frac{1}{\pi_x} & 0 & \frac{1}{\pi_x} & 0 & \frac{1}{\pi_x} \\ \frac{1}{\pi_x} & 0 & \frac{1}{\pi_x} + \frac{1}{\pi_\varepsilon} & 0 & \frac{1}{\pi_x} & 0 \\ 0 & \frac{1}{\pi_x} & 0 & \frac{1}{\pi_x} + \frac{1}{\pi_\varepsilon} & 0 & \frac{1}{\pi_x} \\ \frac{1}{\pi_x} & 0 & \frac{1}{\pi_x} & 0 & \frac{1}{\pi_x} + \frac{\tau^2}{\lambda^2\pi_z\pi_\varepsilon^2} & 0 \\ 0 & \frac{1}{\pi_x} & 0 & \frac{1}{\pi_x} & 0 & \frac{1}{\pi_x} + \frac{\tau^2}{\lambda^2\pi_z\pi_\varepsilon^2} \end{bmatrix} \tag{9}$$

In what follows I will take each investor type separately

Investors $i\,(1,2)$

The information set for $i\,(1,2)$ investors is $F_{i(1,2)} = \{Y_{i1} \ Y_{i2} \ Q_1 \ Q_2\}$. Then

$$\mu_{i(1,2)} \equiv E\left[X \mid F_{i(1,2)}\right] = K_{i(1,2)}^{-1} \begin{bmatrix} \mu_x\pi_x + \frac{\lambda\mu_z\pi_z\pi_\varepsilon}{\tau} + \pi_\varepsilon Y_{i1} + \frac{\lambda^2\pi_z\pi_\varepsilon^2}{\tau^2} Q_1 \\ \mu_x\pi_x + \frac{\lambda\mu_z\pi_z\pi_\varepsilon}{\tau} + \pi_\varepsilon Y_{i2} + \frac{\lambda^2\pi_z\pi_\varepsilon^2}{\tau^2} Q_2 \end{bmatrix} \tag{10}$$

$$K_{i(1,2)} \equiv \mathrm{Var}^{-1}\left[X \mid F_{i(1,2)}\right] = \begin{bmatrix} \pi_x + \pi_\varepsilon + \frac{\lambda^2\pi_z\pi_\varepsilon^2}{\tau^2} & 0 \\ 0 & \pi_x + \pi_\varepsilon + \frac{\lambda^2\pi_z\pi_\varepsilon^2}{\tau^2} \end{bmatrix} \tag{11}$$

Investors $i\,(1)$

The information set for $i\,(1)$ investors is $F_{i(1,2)} = \{Y_{i1}\ Q_1\ Q_2\}$. Then

$$\mu_{i(1)} \equiv E\left[X \mid F_{i(1)}\right] = K_{i(1)}^{-1} \begin{bmatrix} \mu_x\pi_x + \frac{\lambda\mu_z\pi_z\pi_\varepsilon}{\tau} + \pi_\varepsilon Y_{i1} + \frac{\lambda^2\pi_z\pi_\varepsilon^2}{\tau^2}Q_1 \\ \mu_x\pi_x + \frac{\lambda\mu_z\pi_z\pi_\varepsilon}{\tau} + \frac{\lambda^2\pi_z\pi_\varepsilon^2}{\tau^2}Q_2 \end{bmatrix} \tag{12}$$

$$K_{i(1)} \equiv \mathrm{Var}^{-1}\left[X \mid F_{i(1)}\right] = \begin{bmatrix} \pi_x + \pi_\varepsilon + \frac{\lambda^2\pi_z\pi_\varepsilon^2}{\tau^2} & 0 \\ 0 & \pi_x + \frac{\lambda^2\pi_z\pi_\varepsilon^2}{\tau^2} \end{bmatrix} \tag{13}$$

Investors $i\,(2)$

The information set for $i\,(2)$ investors is $F_{i(1,2)} = \{Y_{i2}\ Q_1\ Q_2\}$. Then

$$\mu_{i(2)} \equiv E\left[X \mid F_{i(2)}\right] = K_{i(2)}^{-1} \begin{bmatrix} \mu_x\pi_x + \frac{\lambda\mu_z\pi_z\pi_\varepsilon}{\tau} + \frac{\lambda^2\pi_z\pi_\varepsilon^2}{\tau^2}Q_1 \\ \mu_x\pi_x + \frac{\lambda\mu_z\pi_z\pi_\varepsilon}{\tau} + \pi_\varepsilon Y_{i2} + \frac{\lambda^2\pi_z\pi_\varepsilon^2}{\tau^2}Q_2 \end{bmatrix} \tag{14}$$

$$K_{i(2)} \equiv \mathrm{Var}^{-1}\left[X \mid F_{i(2)}\right] = \begin{bmatrix} \pi_x + \frac{\lambda^2\pi_z\pi_\varepsilon^2}{\tau^2} & 0 \\ 0 & \pi_x + \pi_\varepsilon + \frac{\lambda^2\pi_z\pi_\varepsilon^2}{\tau^2} \end{bmatrix} \tag{15}$$

Investors $i\,(0)$

The information set for $i\,(0)$ investors is $F_{i(1,2)} = \{Q_1\ Q_2\}$. Then

$$\mu_{i(0)} \equiv E\left[X \mid F_{i(0)}\right] = K_{i(0)}^{-1} \begin{bmatrix} \mu_x\pi_x + \frac{\lambda\mu_z\pi_z\pi_\varepsilon}{\tau} + \frac{\lambda^2\pi_z\pi_\varepsilon^2}{\tau^2}Q_1 \\ \mu_x\pi_x + \frac{\lambda\mu_z\pi_z\pi_\varepsilon}{\tau} + \frac{\lambda^2\pi_z\pi_\varepsilon^2}{\tau^2}Q_2 \end{bmatrix} \tag{16}$$

$$K_{i(0)} \equiv \mathrm{Var}^{-1}\left[X \mid F_{i(0)}\right] = \begin{bmatrix} \pi_x + \frac{\lambda^2\pi_z\pi_\varepsilon^2}{\tau^2} & 0 \\ 0 & \pi_x + \frac{\lambda^2\pi_z\pi_\varepsilon^2}{\tau^2} \end{bmatrix} \tag{17}$$

Each investor will have an optimal demand, according to his information set. Then, the optimal demands will be aggregated to impose market clearing and everything is found in closed form with the method of undetermined coefficients. The market clearing condition is

$$\begin{aligned} Z = {} & \left(\lambda^2 - \omega^2\right) S_{i(1,2)} + \left(\lambda - \lambda^2 + \omega^2\right) S_{i(1)} + \left(\lambda - \lambda^2 + \omega^2\right) S_{i(2)} + \\ & + \left((1-\lambda)^2 - \omega^2\right) S_{i(0)} \end{aligned} \tag{18}$$

For the coefficients of the price vector I obtain:

$$a = \begin{bmatrix} \frac{\tau(\lambda\mu_z\pi_z\pi_\varepsilon+\mu_x\pi_x\tau)}{\pi_x\tau^2+\lambda\pi_\varepsilon(\lambda\pi_z\pi_\varepsilon+\tau^2)} \\ \frac{\tau(\lambda\mu_z\pi_z\pi_\varepsilon+\mu_x\pi_x\tau)}{\pi_x\tau^2+\lambda\pi_\varepsilon(\lambda\pi_z\pi_\varepsilon+\tau^2)} \end{bmatrix},\ c = \begin{bmatrix} \frac{\lambda\pi_\varepsilon\left(\lambda\pi_z\pi_\varepsilon+\tau^2\right)}{\pi_x\tau^2+\lambda\pi_\varepsilon(\lambda\pi_z\pi_\varepsilon+\tau^2)} & 0 \\ 0 & \frac{\lambda\pi_\varepsilon\left(\lambda\pi_z\pi_\varepsilon+\tau^2\right)}{\pi_x\tau^2+\lambda\pi_\varepsilon(\lambda\pi_z\pi_\varepsilon+\tau^2)} \end{bmatrix} \tag{19}$$

and

$$B = \begin{bmatrix} \frac{\tau}{\lambda\pi_\varepsilon} & 0 \\ 0 & \frac{\tau}{\lambda\pi_\varepsilon} \end{bmatrix} \tag{20}$$

For the demand coefficients I obtain:

$$\begin{gathered} \alpha = \begin{bmatrix} \frac{(\lambda-1)\pi_\varepsilon(\lambda\mu_z\pi_z\pi_\varepsilon+\mu_x\pi_x\tau)}{\pi_x\tau^2+\lambda\pi_\varepsilon(\lambda\pi_z\pi_\varepsilon+\tau^2)} \\ \frac{(\lambda-1)\pi_\varepsilon(\lambda\mu_z\pi_z\pi_\varepsilon+\mu_x\pi_x\tau)}{\pi_x\tau^2+\lambda\pi_\varepsilon(\lambda\pi_z\pi_\varepsilon+\tau^2)} \end{bmatrix},\ \beta = \begin{bmatrix} \frac{\pi_\varepsilon}{\tau} & 0 \\ 0 & \frac{\pi_\varepsilon}{\tau} \end{bmatrix} \\ \delta = \begin{bmatrix} \frac{\lambda\pi_\varepsilon\left(\lambda\pi_z\pi_\varepsilon^2+(\pi_x+\pi_\varepsilon)\tau^2\right)}{\lambda^2\pi_z\pi_\varepsilon^2\tau+(\pi_x+\lambda\pi_\varepsilon)\tau^3} & 0 \\ 0 & \frac{\lambda\pi_\varepsilon\left(\lambda\pi_z\pi_\varepsilon^2+(\pi_x+\pi_\varepsilon)\tau^2\right)}{\lambda^2\pi_z\pi_\varepsilon^2\tau+(\pi_x+\lambda\pi_\varepsilon)\tau^3} \end{bmatrix} \end{gathered} \tag{21}$$

$$\begin{gathered} \phi_1 = \begin{bmatrix} \frac{(\lambda-1)\pi_\varepsilon(\lambda\mu_z\pi_z\pi_\varepsilon+\mu_x\pi_x\tau)}{\pi_x\tau^2+\lambda\pi_\varepsilon(\lambda\pi_z\pi_\varepsilon+\tau^2)} \\ \frac{\lambda\pi_\varepsilon(\lambda\mu_z\pi_z\pi_\varepsilon+\mu_x\pi_x\tau)}{\pi_x\tau^2+\lambda\pi_\varepsilon(\lambda\pi_z\pi_\varepsilon+\tau^2)} \end{bmatrix},\ \eta_1 = \begin{bmatrix} \frac{\pi_\varepsilon}{\tau} & 0 \\ 0 & 0 \end{bmatrix} \\ \kappa_1 = \begin{bmatrix} \frac{\lambda\pi_\varepsilon\left(\lambda\pi_z\pi_\varepsilon^2+(\pi_x+\pi_\varepsilon)\tau^2\right)}{\lambda^2\pi_z\pi_\varepsilon^2\tau+(\pi_x+\lambda\pi_\varepsilon)\tau^3} & 0 \\ 0 & \frac{\lambda\pi_x\pi_\varepsilon\tau}{\pi_x\tau^2+\lambda\pi_\varepsilon(\lambda\pi_z\pi_\varepsilon+\tau^2)} \end{bmatrix} \end{gathered} \tag{22}$$

$$\begin{gathered} \phi_2 = \begin{bmatrix} \frac{\lambda\pi_\varepsilon(\lambda\mu_z\pi_z\pi_\varepsilon+\mu_x\pi_x\tau)}{\pi_x\tau^2+\lambda\pi_\varepsilon(\lambda\pi_z\pi_\varepsilon+\tau^2)} \\ \frac{(\lambda-1)\pi_\varepsilon(\lambda\mu_z\pi_z\pi_\varepsilon+\mu_x\pi_x\tau)}{\pi_x\tau^2+\lambda\pi_\varepsilon(\lambda\pi_z\pi_\varepsilon+\tau^2)} \end{bmatrix},\ \eta_2 = \begin{bmatrix} 0 & 0 \\ 0 & \frac{\pi_\varepsilon}{\tau} \end{bmatrix} \\ \kappa_2 = \begin{bmatrix} \frac{\lambda\pi_x\pi_\varepsilon\tau}{\pi_x\tau^2+\lambda\pi_\varepsilon(\lambda\pi_z\pi_\varepsilon+\tau^2)} & 0 \\ 0 & \frac{\lambda\pi_\varepsilon\left(\lambda\pi_z\pi_\varepsilon^2+(\pi_x+\pi_\varepsilon)\tau^2\right)}{\lambda^2\pi_z\pi_\varepsilon^2\tau+(\pi_x+\lambda\pi_\varepsilon)\tau^3} \end{bmatrix} \end{gathered} \tag{23}$$

$$\begin{gathered} \zeta = \begin{bmatrix} \frac{\lambda\pi_\varepsilon(\lambda\mu_z\pi_z\pi_\varepsilon+\mu_x\pi_x\tau)}{\pi_x\tau^2+\lambda\pi_\varepsilon(\lambda\pi_z\pi_\varepsilon+\tau^2)} \\ \frac{\lambda\pi_\varepsilon(\lambda\mu_z\pi_z\pi_\varepsilon+\mu_x\pi_x\tau)}{\pi_x\tau^2+\lambda\pi_\varepsilon(\lambda\pi_z\pi_\varepsilon+\tau^2)} \end{bmatrix}, \\ \nu = \begin{bmatrix} \frac{\lambda\pi_x\pi_\varepsilon\tau}{\pi_x\tau^2+\lambda\pi_\varepsilon(\lambda\pi_z\pi_\varepsilon+\tau^2)} & 0 \\ 0 & \frac{\lambda\pi_x\pi_\varepsilon\tau}{\pi_x\tau^2+\lambda\pi_\varepsilon(\lambda\pi_z\pi_\varepsilon+\tau^2)} \end{bmatrix} \end{gathered} \tag{24}$$

A.2 Solution for the Relative Wealth Concerns case

The demand for all investor types are

$$\begin{aligned}
\theta^H_{i(1,2)} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} S_{i(1,2)} + \begin{bmatrix} \psi_H & 0 \\ 0 & \psi_F \end{bmatrix} S_{i(0)} \\
\theta^F_{i(1,2)} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} S_{i(1,2)} + \begin{bmatrix} \psi_F & 0 \\ 0 & \psi_H \end{bmatrix} S_{i(0)} \\
\theta^H_{i(1)} &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} S_{i(1)} + \begin{bmatrix} \psi_{HH} & 0 \\ 0 & \psi_{uH} \end{bmatrix} S_{i(0)} \\
\theta^F_{i(1)} &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} S_{i(1)} + \begin{bmatrix} \psi_{FF} & 0 \\ 0 & \psi_{uF} \end{bmatrix} S_{i(0)} \qquad (25) \\
\theta^H_{i(2)} &= \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} S_{i(2)} + \begin{bmatrix} \psi_{uF} & 0 \\ 0 & \psi_{FF} \end{bmatrix} S_{i(0)} \\
\theta^F_{i(2)} &= \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} S_{i(2)} + \begin{bmatrix} \psi_{uH} & 0 \\ 0 & \psi_{HH} \end{bmatrix} S_{i(0)} \\
\theta^H_{i(0)} = \theta^F_{i(0)} &= \begin{bmatrix} \psi_u & 0 \\ 0 & \psi_u \end{bmatrix} S_{i(0)}
\end{aligned}$$

The optimal demands will be aggregated to impose market clearing as in the benchmark case. The unknown coefficients are now ψ_H, ψ_F, ψ_{HH}, ψ_{uH}, ψ_{FF}, ψ_{uF} and ψ_u. The solutions are complicated functions of the initial parameters, available upon request.

[TO BE COMPLETED]

References

Anat R Admati. A noisy rational expectations equilibrium for multi-asset securities markets. *Econometrica*, 53(3):629–57, May 1985.

Kee-Hong Bae, René M. Stulz, and Hongping Tan. Do local analysts know more? a cross-country study of the performance of local analysts and foreign analysts. *Journal of Financial Economics*, 88(3):581–606, June 2008.

Michael J Brennan and H Henry Cao. International portfolio investment flows. *Journal of Finance*, 52(5):1851–80, December 1997.

Jeffrey R. Brown, Zoran Ivkovic, Paul A. Smith, and Scott Weisbenner. Neighbors matter: Causal community effects and stock market participation. *Journal of Finance*, 63(3):1509–1531, 06 2008.

Vicentiu Covrig, Kalok Chan, and Lilian K. Ng. What Determines the Domestic Bias and Foreign Bias? Evidence from Equity Mutual Fund Allocations Worldwide. *SSRN eLibrary*, 2004.

Lei Feng and Mark S. Seasholes. Correlated trading and location. *Journal of Finance*, 59(5):2117–2144, October 2004.

Diego Garcia and Günter Strobl. Relative Wealth Concerns and Complementarities in Information Acquisition. *SSRN eLibrary*, 2009.

Thomas Gehrig. An information based explanation of the domestic bias in international equity investment. *Scandinavian Journal of Economics*, 95 (1):97–109, 1993.

Juan P. Gomez Lopez, Richard Priestley, and Fernando Zapatero. Implications of Keeping up with the Joneses Behavior for the Equilibrium Cross Section of Stock Returns: International Evidence. *Journal of Finance, Forthcoming*, 2009.

Mark Grinblatt and Matti Keloharju. How distance, language, and culture influence stockholdings and trades. *Journal of Finance*, 56(3):1053–1073, 06 2001.

Harald Hau and Helene Rey. Home bias at the fund level. *American Economic Review*, 98(2):333–38, May 2008.

Harrison Hong, Jeffrey D. Kubik, and Jeremy C. Stein. Social interaction and stock-market participation. *Journal of Finance*, 59(1):137–163, 02 2004.

Zoran Ivkovic and Scott Weisbenner. Local does as local is: Information content of the geography of individual investors' common stock investments. *Journal of Finance*, 60(1):267–306, 02 2005.

Beni Lauterbach and Haim Reisman. Keeping up with the joneses and the home bias. *European Financial Management*, 10(2):225–234, 2004.

Richard Portes and Helene Rey. The determinants of cross-border equity flows. *Journal of International Economics*, 65(2):269–296, March 2005.

Rohit Rahi and José M. Marín. Speculative securities. *Economic Theory*, 14 (3):653–668, 1999.

Piet Sercu and R Vanpee. Home bias in international equity portfolios: a review. Open access publications from katholieke universiteit leuven, Katholieke Universiteit Leuven, 2007.

Stephen H. Shore and Joshua S. White. External Habit Formation and the Home Bias Puzzle. *SSRN eLibrary*, 2002. doi: 10.2139/ssrn.334943.